Assumption Tracking for Optimistic Optimizations

Johannes Doerfert [1] Tobias Grosser [2] Sebastian Pop [3]

[1]Saarbrücken Graduate School of Computer Science
Saarland University
Saarbrücken, Germany

[2]Department of Computer Science
ETH Zürich
Zürich, Switzerland

[3]Samsung Austin R&D Center
Austin, TX, USA

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GRADUATE SCHOOL OF COMPUTER SCIENCE

SPEC 2006 — hmmer — fast_algorithms.c

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```
for (k = 1; k <= M; k++) {
  mc[k] = mpp[k - 1] + tpmm[k - 1];
  if ((sc = ip[k - 1] + tpim[k - 1]) > mc[k]) mc[k] = sc;
  if ((sc = dpp[k - 1] + tpdm[k - 1]) > mc[k]) mc[k] = sc;
  if ((sc = xmb + bp[k]) > mc[k]) mc[k] = sc;
  mc[k] += ms[k];
  if (mc[k] < -INFTY) mc[k] = -INFTY;


  dc[k] = dc[k - 1] + tpdd[k - 1];
  if ((sc = mc[k - 1] + tpmd[k - 1]) > dc[k]) dc[k] = sc;
  if (dc[k] < -INFTY) dc[k] = -INFTY;



  if (k < M) {
    ic[k] = mpp[k] + tpmi[k];
    if ((sc = ip[k] + tpii[k]) > ic[k]) ic[k] = sc;
    ic[k] += is[k];
    if (ic[k] < -INFTY) ic[k] = -INFTY;
  }
}
```

SPEC 2006 — hmmer — fast_algorithms.c


```
#pragma clang loop vectorize(enable)
for (k = 1; k <= M; k++) {
  mc[k] = mpp[k - 1] + tpmm[k - 1];
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}
for (k = 1; k <= M; k++) {
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  }
}
```

+ up to 30% speedup

SPEC 2006 — hmmer — fast_algorithms.c


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    ic[k] += is[k];
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    ic[k] += is[k];
    if (ic[k] < -INFTY) ic[k] = -INFTY;
  }
}
```

+ up to 50% speedup

SPEC 2006 — hmmer — fast_algorithms.c

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1 vectorized loop $\Longrightarrow$ + up to 30% speedup

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1 vectorized loop	$\Longrightarrow$	+	up to 30% speedup
2 vectorized loops	$\Longrightarrow$	+	up to 50% speedup

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1 vectorized loop	$\Longrightarrow$	+	up to 30% speedup
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possible aliasing	$\Longrightarrow$	-	runtime alias checks

SPEC 2006 — hmmer — fast_algorithms.c

1 vectorized loop	$\Longrightarrow$	+ up to 30% speedup
2 vectorized loops	$\Longrightarrow$	+ up to 50% speedup
possible aliasing	$\Longrightarrow$	- runtime alias checks
possible dependences	$\Longrightarrow$	- static dependence analysis

PARSEC — blackscholes — blackscholes.c

PARSEC — blackscholes — blackscholes.c


```c
float BlkSchlsEqEuroNoDiv(float sptprice, float strike, float rate,
                          float volatility, float time, int otype) {
    float xD1, xD2, xDen, d1, d2, FutureValueX, NofXd1, NofXd2, NegNofXd1,
          NegNofXd2, Price;
    xD1 = rate + volatility * volatility; * 0.5;
    xD1 = xD1 * time;
    xD1 = xD1 + log( sptprice / strike );
    xDen = volatility * sqrt(time);
    xD1 = xD1 / xDen;
    xD2 = xD1 -  xDen;
    d1 = xD1;
    d2 = xD2;
    NofXd1 = CNDF( d1 );
    NofXd2 = CNDF( d2 );
    FutureValueX = strike * ( exp( -(rate)*(time) ) );
    if (otype == 0) {
        Price = (sptprice * NofXd1) - (FutureValueX * NofXd2);
    } else {
        NegNofXd1 = (1.0 - NofXd1);
        NegNofXd2 = (1.0 - NofXd2);
        Price = (FutureValueX * NegNofXd2) - (sptprice * NegNofXd1);
    }
    return Price;
}
```

PARSEC — blackscholes — blackscholes.c

```
int bs_thread(void *tid_ptr) {
    int tid = *(int *)tid_ptr;
    int start = tid * (numOptions / nThreads);
    int end = start + (numOptions / nThreads);

    for (int j = 0; j < NUM_RUNS; j++)
        for (int i = start; i < end; i++)
          prices[i] = BlkSchlsEqEuroNoDiv(sptprice[i], strike[i], rate[i],
                                          volatility[i], otime[i], otype[i]);
    return 0;
}
```

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    return 0;
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```

- + 2.9× speedup for manual parallelization on a quad-core i7

PARSEC — blackscholes — blackscholes.c

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- \- Possible aliasing arrays

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- Possible aliasing arrays
- Possible execution of non-pure calls

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+ 2.9× speedup for manual parallelization on a quad-core i7
+ 2.8× speedup for automatic parallelization on a quad-core i7

- Possible aliasing arrays
- Possible execution of non-pure calls
- Possible execution of dead-iterations (`0 <= j < NUM_RUNS - 1`)

PARSEC — blackscholes — blackscholes.c

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    int start = tid * (numOptions / nThreads);
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    for (int j = 0; j < NUM_RUNS; j++)
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          prices[i] = BlkSchlsEqEuroNoDiv(sptprice[i], strike[i], rate[i],
                                          volatility[i], otime[i], otype[i]);
    return 0;
}
```

- \+ 2.9× speedup for manual parallelization on a quad-core i7
- \+ 2.8× speedup for automatic parallelization on a quad-core i7
- \+ 6.5× speedup for sequential execution (native input)
- \- Possible aliasing arrays
- \- Possible execution of non-pure calls
- \- Possible execution of dead-iterations (`0 <= j < NUM_RUNS - 1`)

NAS Parallel Benchmarks — BT — rhs.c

NAS Parallel Benchmarks — BT — rhs.c


```
void compute_rhs() {
  int i, j, k, m;
  double rho_inv, uijk, up1, um1, vijk, vp1, vm1, wijk, wp1, wm1;

  if (timeron) timer_start(t_rhs);

  for (k = 0; k <= grid_points[2]-1; k++) {
    for (j = 0; j <= grid_points[1]-1; j++) {
      for (i = 0; i <= grid_points[0]-1; i++) {
        rho_inv = 1.0/u[k][j][i][0];
        rho_i[k][j][i] = rho_inv;
        us[k][j][i] = u[k][j][i][1] * rho_inv;
        vs[k][j][i] = u[k][j][i][2] * rho_inv;
        ws[k][j][i] = u[k][j][i][3] * rho_inv;
        square[k][j][i] = 0.5* (
            u[k][j][i][1]*u[k][j][i][1] +
            u[k][j][i][2]*u[k][j][i][2] +
            u[k][j][i][3]*u[k][j][i][3] ) * rho_inv;
        qs[k][j][i] = square[k][j][i] * rho_inv;
      }
    }
  }
```

NAS Parallel Benchmarks — BT — rhs.c

```
for (k = 0; k <= grid_points[2]-1; k++) {
  for (j = 0; j <= grid_points[1]-1; j++) {
    for (i = 0; i <= grid_points[0]-1; i++) {
      for (m = 0; m < 5; m++) {
        rhs[k][j][i][m] = forcing[k][j][i][m];
      }
    }
  }
}

if (timeron) timer_start(t_rhsx);

for (k = 1; k <= grid_points[2]-2; k++) {
  for (j = 1; j <= grid_points[1]-2; j++) {
    for (i = 1; i <= grid_points[0]-2; i++) {
      uijk = us[k][j][i];
      up1  = us[k][j][i+1];
      um1  = us[k][j][i-1];

      rhs[k][j][i][0] = rhs[k][j][i][0] + dx1tx1 *
        (u[k][j][i+1][0] - 2.0*u[k][j][i][0] +
          u[k][j][i-1][0]) -
        tx2 * (u[k][j][i+1][1] - u[k][j][i-1][1]);
```

NAS Parallel Benchmarks — BT — rhs.c


```
rhs[k][j][i][1] = rhs[k][j][i][1] + dx2tx1 *
  (u[k][j][i+1][1] - 2.0*u[k][j][i][1] +
   u[k][j][i-1][1]) +
  xxcon2*con43 * (up1 - 2.0*uijk + um1) -
  tx2 * (u[k][j][i+1][1]*up1 -
      u[k][j][i-1][1]*um1 +
      (u[k][j][i+1][4]- square[k][j][i+1]-
       u[k][j][i-1][4]+ square[k][j][i-1])* c2);

rhs[k][j][i][2] = rhs[k][j][i][2] + dx3tx1 *
  (u[k][j][i+1][2] - 2.0*u[k][j][i][2] +
   u[k][j][i-1][2]) +
  xxcon2 * (vs[k][j][i+1] - 2.0*vs[k][j][i] +
      vs[k][j][i-1]) -
  tx2 * (u[k][j][i+1][2]*up1 - u[k][j][i-1][2]*um1);

rhs[k][j][i][3] = rhs[k][j][i][3] + dx4tx1 *
  (u[k][j][i+1][3] - 2.0*u[k][j][i][3] +
   u[k][j][i-1][3]) +
  xxcon2 * (ws[k][j][i+1] - 2.0*ws[k][j][i] +
      ws[k][j][i-1]) -
  tx2 * (u[k][j][i+1][3]*up1 - u[k][j][i-1][3]*um1);

  /* ≈300 more lines of similar code */
```

NAS Parallel Benchmarks — BT — rhs.c


```
for (k = 0; k <= grid_points[2]-1; k++)
  for (j = 0; j <= grid_points[1]-1; j++)
    for (i = 0; i <= grid_points[0]-1; i++)
      for (m = 0; m < 5; m++)
        rhs[k][j][i][m] = forcing[k][j][i][m];

if (timeron) timer_start(t_rhsx);

for (k = 1; k <= grid_points[2]-2; k++) {
  for (j = 1; j <= grid_points[1]-2; j++) {
    for (i = 1; i <= grid_points[0]-2; i++) {
      /* ... */
```

[a] Sanyam and Yew, PLDI 15

NAS Parallel Benchmarks — BT — rhs.c

```
for (k = 0; k <= grid_points[2]-1; k++)
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    for (i = 0; i <= grid_points[0]-1; i++)
      for (m = 0; m < 5; m++)
        rhs[k][j][i][m] = forcing[k][j][i][m];

if (timeron) timer_start(t_rhsx);

for (k = 1; k <= grid_points[2]-2; k++) {
  for (j = 1; j <= grid_points[1]-2; j++) {
    for (i = 1; i <= grid_points[0]-2; i++) {
      /* ... */
```

- + 6× speedup for 8 threads/cores [a]

[a] Sanyam and Yew, PLDI 15

NAS Parallel Benchmarks — BT — rhs.c


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for (k = 0; k <= grid_points[2]-1; k++)
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        rhs[k][j][i][m] = forcing[k][j][i][m];

if (timeron) timer_start(t_rhsx);

for (k = 1; k <= grid_points[2]-2; k++) {
  for (j = 1; j <= grid_points[1]-2; j++) {
    for (i = 1; i <= grid_points[0]-2; i++) {
      /* ... */
```

- \+ 6× speedup for 8 threads/cores [a]
- \- Possible variant loop bounds

[a] Sanyam and Yew, PLDI 15

NAS Parallel Benchmarks — BT — rhs.c

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for (k = 0; k <= grid_points[2]-1; k++)
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      for (m = 0; m < 5; m++)
        rhs[k][j][i][m] = forcing[k][j][i][m];

if (timeron) timer_start(t_rhsx);

for (k = 1; k <= grid_points[2]-2; k++) {
  for (j = 1; j <= grid_points[1]-2; j++) {
    for (i = 1; i <= grid_points[0]-2; i++) {
      /* ... */
```

- \+ $6\times$ speedup for 8 threads/cores [a]
- \- Possible variant loop bounds
- \- Possible out-of-bound accesses

[a] Sanyam and Yew, PLDI 15

NAS Parallel Benchmarks — BT — rhs.c


```
for (k = 0; k <= grid_points[2]-1; k++)
  for (j = 0; j <= grid_points[1]-1; j++)
    for (i = 0; i <= grid_points[0]-1; i++)
      for (m = 0; m < 5; m++)
        rhs[k][j][i][m] = forcing[k][j][i][m];

if (timeron) timer_start(t_rhsx);

for (k = 1; k <= grid_points[2]-2; k++) {
  for (j = 1; j <= grid_points[1]-2; j++) {
    for (i = 1; i <= grid_points[0]-2; i++) {
      /* ... */
```

- + 6× speedup for 8 threads/cores [a]
- - Possible variant loop bounds
- - Possible out-of-bound accesses
- - Possible execution of non-pure calls

[a] Sanyam and Yew, PLDI 15

NAS Parallel Benchmarks — BT — rhs.c

```
for (k = 0; k <= grid_points[2]-1; k++)
  for (j = 0; j <= grid_points[1]-1; j++)
    for (i = 0; i <= grid_points[0]-1; i++)
      for (m = 0; m < 5; m++)
        rhs[k][j][i][m] = forcing[k][j][i][m];

if (timeron) timer_start(t_rhsx);

for (k = 1; k <= grid_points[2]-2; k++) {
  for (j = 1; j <= grid_points[1]-2; j++) {
    for (i = 1; i <= grid_points[0]-2; i++) {
      /* ... */
```

- \+ 6× speedup for 8 threads/cores [a]
- \- Possible variant loop bounds
- \- Possible out-of-bound accesses
- \- Possible execution of non-pure calls
- \- Possible integer under/overflows complicate loop bounds

[a] Sanyam and Yew, PLDI 15

Be Optimistic!

Programs might be nasty but *programmers* are not.

When static information is insufficient

Optimistic Assumptions & Speculative Versioning

When static information is insufficient

Optimistic Assumptions & Speculative Versioning

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Optimistic Assumptions

1. Take optimistic assumptions to (better) optimize loops

When static information is insufficient

Optimistic Assumptions & Speculative Versioning

Optimistic Assumptions

1. Take optimistic assumptions to (better) optimize loops
2. Derive simple runtime conditions that imply these assumptions

When static information is insufficient

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Optimistic Assumptions

1. Take optimistic assumptions to (better) optimize loops
2. Derive simple runtime conditions that imply these assumptions
3. Version the code based on the assumptions made and conditions derived.

When static information is insufficient

Optimistic Assumptions & Speculative Versioning

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Optimistic Assumptions

1. Take optimistic assumptions to (better) optimize loops
2. Derive simple runtime conditions that imply these assumptions
3. Version the code based on the assumptions made and conditions derived.

Speculative Versioning

```
if (/* Runtime Conditions */)
  /* Optimized Loop Nest */
else
  /* Original Loop Nest */
```

When static information is insufficient

Runtime Conditions

Runtime Conditions

- Fast to derive (compile time)
- Fast to verify (runtime)
- High probability to be true

Polly

The polyhedral loop optimizer in LLVM

Polly

The polyhedral loop optimizer in LLVM

Polyhedral Optimizer

- Loop Nest Optimizer
- Precise compute model (affine constraints)
- Combines many classical loop optimizations
  - Tiling, Interchange, Fusion, Fission, . . .
- Examples: Pluto, ppcg

Polly

The polyhedral loop optimizer in LLVM

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Polly — Advantages

- *Automatic & Semi-automatic Mode*
- *Robust* & *Widely Applicable*
- Embedded in LLVM
  - Source & Target Independent
  - Information flow between Polly and other passes

Optimistic Assumptions in Polly

Possibly Invariant Loads

```c
void loop_bounds(int *size0, int *size1) {




  for (int i = 0; i < *size0; i++)
    for (int j = 0; j < *size1; j++)
      ...
}
```

Optimistic Assumptions in Polly

Possibly Invariant Loads

```
void loop_bounds(int *size0, int *size1) {
  int size0val = *size0;
  int size1val = *size1;



  for (int i = 0; i < size0val; i++)
    for (int j = 0; j < size1val; j++)
      ...
}
```

- Hoist invariant loads

Optimistic Assumptions in Polly

Possibly Invariant Loads


```
void loop_bounds(int *size0, int *size1) {
  int size0val = *size0;
  int size1val = 0;

  if (size0val > 0)
    size1val = *size1;

  for (int i = 0; i < size0val; i++)
    for (int j = 0; j < size1val; j++)
      ...
}
```

- Hoist invariant loads
- Keep conditions for conditionally executed loads

Optimistic Assumptions in Polly

Possibly Invariant Loads

```
void loop_bounds(int *size0, int *size1) {
  int size0val = *size0;
  int size1val = 0;

  if (size0val > 0)
    size1val = *size1;

  for (int i = 0; i < size0val; i++)
    for (int j = 0; j < size1val; j++)
      ...
}
```

- Hoist invariant loads
- Keep conditions for conditionally executed loads
- Powerful in combination with alias checks

Loop Optimizations Today and with Polly

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	Today in LLVM	Polly
Target	Inner Loops	(Unstructured) Loops Nests

Loop Optimizations Today and with Polly

	Today in LLVM	**Polly**
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Approach	Seq. of Specialized Passes	Combined Modeling

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Optimistic Assumptions in Polly

(A) Applicability/Correctness

1. No Alias Assumption[1]
2. No Wrapping Assumption[2]
3. Finite Loops Assumption[2]
4. Array In-bounds Assumption[2]
5. Valid Multidimensional View Assumption (Delinearization)[3]
6. Possibly Invariant Loads

(B) Optimizations

1. Array In-bounds Check Hoisting[2]
2. Parametric Dependence Distances[4]

---

[1] Joint work Fabrice Rastello (INRIA Grenoble) & others. [OOPSLA'15]

[2] Joint work with Tobias Grosser (ETH)

[3] Tobias Grosser & Sebastian Pop (Samsung) [ICS'15]

[4] Joint work with Zino Benaissa (Qualcomm)

Ongoing Work

- **Polly Mainline**
  - Improved optimization choices
  - Profile guided optimization
  - More powerful checks (e.g., generate inspector loops)
  - User feedback, register tiling, user provided assumptions
  - ...

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  - *Integrate Polly into LLVM mainline*

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  - Improved optimization choices
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- **Independent Projects using Polly**
  - Heterogeneous Compute (OpenCL)
  - High-level Synthesis
  - Dynamic Compilation (JITs)

Ongoing Work

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- **Independent Projects using Polly**
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Thank You.

Polly

Incomplete feature list

- **Loops & Conditions**
  - Styles

    `for/while/do/goto`
  - Arbitrary Presburger Expressions

    ```
    for (i = 0; i < 22 && i < mod(i + b, 13); i += 2)
    if (5 * i + b <= 13 || 12 > b)
    ```
  - Multiple back-edges/exit-edges & unstructured control flow

    `break; continue; goto; switch`
  - Data-dependent control flow

    `if (B[i]) A[i] = A[i] / B[i];`
- **Accesses**
  - Multi-dimensionality: `A[][n][m] / A[][10][100]`
  - Non-affine: `A[i * j]`
  - Scalar: `x = A[i]; ...; B[i] = x;`
- **User Annotations**
  - `__builtin_assume(M > 8)`
  - `__builtin_assume(__polly_profitable == 1)`

NAS Parallel Benchmarks — BT — rhs.c


```
clang -Rpass-analysis=polly-scops -O3 -polly rhs.c
```

```
rhs.c:47:3: remark: SCoP begins here. [-Rpass-analysis=polly-scops]
  for (k = 0; k <= grid_points[2]-1; k++) {
  ^
/* ... */
rhs.c:418:16: remark: SCoP ends here. [-Rpass-analysis=polly-scops]
  if (timeron) timer_stop(t_rhs);
               ^
```

NAS Parallel Benchmarks — BT — rhs.c

```
clang -Rpass-analysis=polly-scops -O3 -polly rhs.c
```

```
rhs.c:79:16: remark: No-error assumption: [grid_points, grid_points', timeron] ->
      {  : timeron = 0 } [-Rpass-analysis=polly-scops]
  if (timeron) timer_start(t_rhsx);
               ^
```

NAS Parallel Benchmarks — BT — rhs.c

`clang -Rpass-analysis=polly-scops -O3 -polly rhs.c`

```
rhs.c:50:23: remark: Inbounds assumption: [grid_points, grid_points', grid_points''] ->
 {  : grid_points <= 0 or (grid_points >= 1 and grid_points' <= 0) or (grid_points >= 1 and
      grid_points' >= 104 and grid_points'' <= 0) or (grid_points >= 1 and grid_points' <= 103
      and grid_points' >= 1 and grid_points'' <= 103) } [-Rpass-analysis=polly-scops]
        rho_inv = 1.0/u[k][j][i][0];
                      ^
rhs.c:144:27: remark: Inbounds assumption: [grid_points, grid_points', timeron, grid_points''] ->
 {  : grid_points <= 2 or (grid_points >= 3 and grid_points' <= 104) } [-Rpass-analysis=polly-scops]
        rhs[k][j][i][m] = rhs[k][j][i][m]- dssp *
                          ^
rhs.c:171:27: remark: Inbounds assumption: [grid_points, grid_points', timeron, grid_points''] ->
 {  : grid_points <= 2 or (grid_points >= 3 and grid_points' <= 2) or (grid_points >= 3
      and grid_points' <= 104 and grid_points' >= 3 and grid_points'' <= 105 and grid_points'' >= 3) }
        rhs[k][j][i][m] = rhs[k][j][i][m] - dssp *
                          ^
```

NAS Parallel Benchmarks — BT — rhs.c

`clang -Rpass-analysis=polly-scops -O3 -polly rhs.c`

```
rhs.c:419:1: remark: No-overflows assumption: [grid_points, grid_points', grid_points'', timeron] ->
{: (grid_points >= 3 and grid_points' >= 3 and grid_points'' >= -2147483643) or (grid_points >= 3 and
   grid_points' <= 2 and grid_points' >= -2147483643 and grid_points'' >= -2147483646) or
   (grid_points <= 2 and grid_points >= -2147483643 and grid_points' >= 3 and grid_points'' >= -2147483646) or
   (grid_points <= 2 and grid_points >= -2147483644 and grid_points' <= 2 and grid_points' >= -2147483646) or
   (grid_points' = -2147483644 and grid_points >= 3 and grid_points'' <= 2 and grid_points'' >= -2147483646) or
   (grid_points = -2147483644 and grid_points' >= 3 and grid_points'' <= 2 and grid_points'' >= -2147483646) }
```

```
__builtin_assume(grid_points[0] >= -2147483643 &&
                 grid_points[1] >= -2147483643 &&
                 grid_points[2] >= -2147483643);
```

NAS Parallel Benchmarks — BT — rhs.c

`clang -Rpass-analysis=polly-scops -O3 -polly rhs.c`

```
rhs.c:50:23: remark: Possibly aliasing pointer, use restrict keyword.
      [-Rpass-analysis=polly-scops]
        rho_inv = 1.0/u[k][j][i][0];
                      ^
rhs.c:56:13: remark: Possibly aliasing pointer, use restrict keyword.
      [-Rpass-analysis=polly-scops]
            u[k][j][i][1]*u[k][j][i][1] +
            ^
```

Optimistic Assumptions in Polly

Array In-bounds Assumptions


```
void stencil(int N, int M, float A[128][128]) {




    for (int i = 0; i < N; i++)
      for (int j = 0; j < M; j++)
        A[j][i] += A[2*j+1][i];

}
```

Optimistic Assumptions in Polly

Array In-bounds Assumptions

```
void stencil(int N, int M, float A[128][128]) {
    if ([redacted]) {
      for (int j = 0; j < M; j++)
        for (int i = 0; i < N; i++)
          A[j][i] += A[2*j+1][i];
    } else {
      for (int i = 0; i < N; i++)
        for (int j = 0; j < M; j++)
          A[j][i] += A[2*j+1][i];
    }
}
```

Optimistic Assumptions in Polly

Array In-bounds Assumptions


```
void stencil(int N, int M, float A[128][128]) {
    if (N < 128) {
      for (int j = 0; j < M; j++)
        for (int i = 0; i < N; i++)
          A[j][i] += A[2*j+1][i];
    } else {
      for (int i = 0; i < N; i++)
        for (int j = 0; j < M; j++)
          A[j][i] += A[2*j+1][i];
    }
}
```

- Out-of-bound accesses introduce multiple addresses for one memory location (e.g., `&A[1][0] == &A[0][128]`)

Optimistic Assumptions in Polly


```
void mem_copy(int N, float *A, float *B) {
  if ([redacted] || [redacted]) {

    #pramga vectorize
    for (i = 0; i < N; i++)
      A[i] = B[i+5];

  } else {
    /* original code */
  }
}
```

Optimistic Assumptions in Polly


```
void mem_copy(int N, float *A, float *B) {
  if (&A[0] >= &B[N+5] || [redacted]) {

    #pramga vectorize
    for (i = 0; i < N; i++)
      A[i] = B[i+5];

  } else {
    /* original code */
  }
}
```

Optimistic Assumptions in Polly

No Alias Assumptions


```
void mem_copy(int N, float *A, float *B) {
  if (&A[0] >= &B[N+5] || &A[N] <= &B[5]) {

    #pramga vectorize
    for (i = 0; i < N; i++)
      A[i] = B[i+5];

  } else {
    /* original code */
  }
}
```

- Compare minimal/maximal accesses to possible aliasing arrays

Optimistic Assumptions in Polly

No Alias Assumptions

```
void evn_odd(int N, int *Evn, int *Odd, int *A, int *B) {
  if ([redacted]) {
    for (int i = 0; i < N; i += 2)
      if (N % 2)
        Odd[i/2] = A[i+1] - B[i+1];
      else
        Evn[i/2] = A[i] + B[i];
  } else {
    /* original code */
  }
}
```

- Compare minimal/maximal accesses to possible aliasing arrays

Optimistic Assumptions in Polly

No Alias Assumptions

```
void evn_odd(int N, int *Evn, int *Odd, int *A, int *B) {
  if ([redacted]) {

    for (int i = 0; i < N; i += 2)
      if (N % 2)
        Odd[i/2] = A[i+1] - B[i+1];
      else
        Evn[i/2] = A[i] + B[i];

  } else {
    /* original code */
  }
}
```

- Compare minimal/maximal accesses to possible aliasing arrays
- Do not compare accesses to read-only arrays

Optimistic Assumptions in Polly

No Alias Assumptions

```
void evn_odd(int N, int *Evn, int *Odd, int *A, int *B) {

  if ([redacted]) {

    for (int i = 0; i < N; i += 2)
      if (N % 2)
        Odd[i/2] = A[i+1] - B[i+1];
      else
        Evn[i/2] = A[i] + B[i];

  } else {
    /* original code */
  }
}
```

- Compare minimal/maximal accesses to possible aliasing arrays
- Do not compare accesses to read-only arrays
- Use the iteration domain of the accesses

Optimistic Assumptions in Polly

No Alias Assumptions

```
void evn_odd(int N, int *Evn, int *Odd, int *A, int *B) {

  if (N%2 ? ((&B[N+1] <= &Odd[0] || &Odd[(N+1)/2] <= &B[1]) &&
              (&A[N+1] <= &Odd[0] || &Odd[(N+1)/2] <= &A[1]))
          : ((&B[N] <= &Evn[0] || &Evn[(N+1)/2] <= &B[0]) &&
              (&A[N] <= &Evn[0] || &Evn[(N+1)/2] <= &A[0])) ) {

    for (int i = 0; i < N; i += 2)
      if (N % 2)
        Odd[i/2] = A[i+1] - B[i+1];
      else
        Evn[i/2] = A[i] + B[i];

  } else {
    /* original code */
  }
}
```

- Compare minimal/maximal accesses to possible aliasing arrays
- Do not compare accesses to read-only arrays
- Use the iteration domain of the accesses

Optimistic Assumptions in Polly

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```
void mem_shift(unsigned char N, float *A) {
  if ([redacted]) {

    #pramga vectorize
    for (unsigned char i = 0; i < N; i++)
      A[i] = A[N + i];

  } else {
    /* original code */
  }
}
```

Optimistic Assumptions in Polly

No Wrapping Assumption


```
void mem_shift(unsigned char N, float *A) {
  if (N <= 128) {

    #pramga vectorize
    for (unsigned char i = 0; i < N; i++)
      A[i] = A[N + i];

  } else {
    /* original code */
  }
}
```

- Finite bit width can cause integer expressions to “wrap around”
- Wrapping causes multiple addresses for one memory location

Optimistic Assumptions in Polly

No Wrapping Assumption

$$\underline{i} * \underline{c_0} + \underline{p} * \underline{c_1} \equiv_p (\underline{i} * \underline{c_0} + \underline{p} * \underline{c_1}) \ \mathtt{mod}\ 2^k$$

Optimistic Assumptions in Polly

No Wrapping Assumption

$$\underline{i} * \underline{c_0} + \underline{p} * \underline{c_1} \;\equiv_p\; (\underline{i} * \underline{c_0} + \underline{p} * \underline{c_1}) \;\texttt{mod}\; 2^k$$

$$\texttt{i} \in [\texttt{0}, \texttt{N-1}] \wedge \texttt{N} \in [\texttt{0}, 2^8]$$

$$(\texttt{N + i}) \equiv_p (\texttt{N + i}) \;\texttt{mod}\; 2^8$$

Optimistic Assumptions in Polly

No Wrapping Assumption

$$\underline{i} * \underline{c_0} + \underline{p} * \underline{c_1} \;\equiv_p\; (\underline{i} * \underline{c_0} + \underline{p} * \underline{c_1}) \;\texttt{mod}\; 2^k$$

$$\texttt{i} \in [\texttt{0}, \texttt{N-1}] \wedge \texttt{N} \in [\texttt{0}, 2^8]$$

$$(\texttt{N + i}) \equiv_p (\texttt{N + i}) \;\texttt{mod}\; 2^8$$

$$\Longrightarrow \quad (\texttt{N + i}) \leq_p \texttt{255}$$

Optimistic Assumptions in Polly

No Wrapping Assumption

$$\underline{i} * \underline{c_0} + \underline{p} * \underline{c_1} \equiv_p (\underline{i} * \underline{c_0} + \underline{p} * \underline{c_1}) \texttt{ mod } 2^k$$

$$\texttt{i} \in \texttt{[0, N-1]} \wedge \texttt{N} \in \texttt{[0, } 2^8\texttt{]}$$

$$\texttt{(N + i)} \equiv_p \texttt{(N + i) mod } 2^8$$

$$\Longrightarrow \texttt{(N + i)} \leq_p \texttt{255}$$

$$\Longrightarrow \texttt{N} \leq \texttt{128}$$

Optimistic Assumptions in Polly


```
void mem_shift(unsigned N, float *A) {
  if ([redacted]) {

    #pramga vectorize
    for (unsigned i = 0; i != N; i+=2)
      A[i+4] = A[i];

  } else {
    /* original code */
  }
}
```

Optimistic Assumptions in Polly

Finite Loops Assumption


```
void mem_shift(unsigned N, float *A) {
  if (N % 2 == 0) {

    #pramga vectorize
    for (unsigned i = 0; i != N; i+=2)
      A[i+4] = A[i];

  } else {
    /* original code */
  }
}
```

- Allows to provide other LLVM passes *real* loop bounds
- Infinite loops create unbounded optimization problems

Optimistic Assumptions in Polly

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```
#define A(x, y) A[n1 * x + y]
void set_subarray(float *A, int o0, int o1, int s0,
                            int s1, int n0, int n1) {
  if ([redacted]) {

    #pragma parallel
    for (int i = 0; i < s0; i++)
      for (int j = 0; j < s1; j++)
        A(o0 + i, o1 + j) = 1;

  } else {
    /* original code */
  }
}
```

Optimistic Assumptions in Polly


```
#define A(x, y) A[n1 * x + y]
void set_subarray(float *A, int o0, int o1, int s0,
                                int s1, int n0, int n1) {
  if ([redacted]) {

    #pragma parallel
    for (int i = 0; i < s0; i++)
      for (int j = 0; j < s1; j++)
        A(o0 + i, o1 + j) = 1;

  } else {
    /* original code */
  }
}
```

Optimistic Assumptions in Polly

Valid Multidimensional View Assumption

```
#define A(x, y) A[n1 * x + y]
void set_subarray(float *A, int o0, int o1, int s0,
                            int s1, int n0, int n1) {
  if (o1 + s1 <= n1) {

    #pragma parallel
    for (int i = 0; i < s0; i++)
      for (int j = 0; j < s1; j++)
        A(o0 + i, o1 + j) = 1;

  } else {
    /* original code */
  }
}
```

- Define multi-dimensional view of a linearized (one-dimensional) array
- Derive conditions that accesses are in-bounds for each dimension

Optimistic Assumptions in Polly

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```
struct SafeArray { int Size, int *Array };

inline void set(SafeArray A, int idx, int val) {
  if (idx < 0 || A.Size <= idx)
    throw OutOfBounds;
  A.Array[idx] = val;
}

void set_safe_array(int N, SafeArray A) {
    for (int i = 0; i < N; i++)
      for (int j = 0; j < i/2; j++)
        set(A, i+j, 1); /* Throws out-of-bounds */
}
```

Optimistic Assumptions in Polly


```
struct SafeArray { int Size, int *Array };

inline void set(SafeArray A, int idx, int val) {
  if (idx < 0 || A.Size <= idx)
    throw OutOfBounds;
  A.Array[idx] = val;
}

void set_safe_array(int N, SafeArray A) {
    for (int i = 0; i < N; i++)
      for (int j = 0; j < i/2; j++)
        set(A, i+j, 1); /* Throws out-of-bounds */
}
```

Optimistic Assumptions in Polly

Array In-bounds Check Hoisting

```
struct SafeArray { int Size, int *Array };

inline void set(SafeArray A, int idx, int val) {
  if (idx < 0 || A.Size <= idx)
    throw OutOfBounds;
  A.Array[idx] = val;
}

void set_safe_array(int N, SafeArray A) {
    for (int i = 0; i < N; i++)
      for (int j = 0; j < i/2; j++)
        set(A, i+j, 1); /* Throws out-of-bounds */
}
```

Optimistic Assumptions in Polly

Array In-bounds Check Hoisting

```
void set_safe_array(int N, SafeArray A) {
    if ([redacted]) {

      for (int i = 0; i < N; i++)
        for (int j = 0; j < i/2; j++)
          A[i+j] = 1;

    } else {
      /* original code */
    }
  }
```

Optimistic Assumptions in Polly

Array In-bounds Check Hoisting

```
void set_safe_array(int N, SafeArray A) {
    if ((3*N)/2 <= A.Size) {

      for (int i = 0; i < N; i++)
        for (int j = 0; j < i/2; j++)
          A[i+j] = 1;

    } else {
      /* original code */
    }
  }
```

- Hoist in-bounds access conditions out of the loop nest

Optimistic Assumptions in Polly

Check Hoisting


```
void copy(int N, double A[N][N], double B[N][N]) {
    if (DebugLevel <= 5) {

      #pragma parallel
      for (int i = 0; i < N; i++)
        #pragma simd
        for (int j = 0; j < N; j++)
          A[i][j] = B[i][j];

    } else {

      for (int i = 0; i < N; i++) {
        for (int j = 0; j < N; j++)
          A[i][j] = B[i][j];

        if (DebugLevel > 5)
          printf("Column␣\%d␣copied\n", i)
      }

    }
  }
```

Optimistic Assumptions in Polly


```
void vectorize(int N, double *A) {
    if ([redacted]) {

      #pragma vectorize width(4)
      for (int i = c; i < N+c; i++)
        A[i-c] += A[i];

    } else {
      /* original code */
    }
  }
```

Optimistic Assumptions in Polly

Parametric Dependence Distances


```
void vectorize(int N, double *A) {
    if (c >= 4) {

      #pragma vectorize width(4)
      for (int i = c; i < N+c; i++)
        A[i-c] += A[i];

    } else {
      /* original code */
    }
  }
```

- Assume *large enough* dependence distance, e.g., for vectorization

Motivation

Motivation

- Modern off-the-shelf processors are complex and powerful
  - low overhead vector units
  - multiple cores and levels of cache

Motivation

- Modern off-the-shelf processors are complex and powerful
  - low overhead vector units
  - multiple cores and levels of cache
- Programs do not exploit this power
  - not cache aware
  - written for single threaded execution

Motivation

```
for (i = 0; i < nx; i++) {
  for (j = 0; j < ny; j++) {
    q[i] = q[i] + A[i][j] * p[j];
    s[j] = s[j] + r[i] * A[i][j];
  }
}
```

Motivation

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```
if ((ny >= 1)) {
  ub1 = floord((nx + -1), 256);
#pragma omp parallel for private(c2, c3, c4, c5, c6) firstprivate(ub1)
  for (c1 = 0; c1 <= ub1; c1++)
    for (c2 = 0; c2 <= floord((ny + -1), 256); c2++)
      for (c3 = (8 * c1); c3 <= min(floord((nx + -1), 32), ((8 * c1) + 7)); c3++)
        for (c4 = (8 * c2); c4 <= min(floord((ny + -1), 32), ((8 * c2) + 7)); c4++)
          for (c5 = (32 * c4); c5 <= min(((32 * c4) + 31), (ny + -1)); c5++)
#pragma ivdep
#pragma vector always
#pragma simd
            for (c6 = (32 * c3); c6 <= min(((32 * c3) + 31), (nx + -1)); c6++)
              q[c6]=q[c6]+A[c6][c5]*p[c5];
}
if ((nx >= 1)) {
  ub1 = floord((ny + -1), 256);
#pragma omp parallel for private(c2, c3, c4, c5, c6) firstprivate(ub1)
  for (c1 = 0; c1 <= ub1; c1++)
    for (c2 = 0; c2 <= floord((nx + -1), 256); c2++)
      for (c3 = (8 * c1); c3 <= min(floord((ny + -1), 32), ((8 * c1) + 7)); c3++)
        for (c4 = (8 * c2); c4 <= min(floord((nx + -1), 32), ((8 * c2) + 7)); c4++)
          for (c5 = (32 * c4); c5 <= min(((32 * c4) + 31), (nx + -1)); c5++)
#pragma ivdep
#pragma vector always
#pragma simd
            for (c6 = (32 * c3); c6 <= min(((32 * c3) + 31), (ny + -1)); c6++)
              s[c6]=s[c6]+r[c5]*A[c5][c6];
}
```

The Polyhedral Model

Input Loop Nest

Polyhedral Abstraction

```
for (i = 0; i <= 1024; i++) {
  for (j = 0; j <= 1024; j++) {
    s[j] = s[j] + r[i] * A[i][j];
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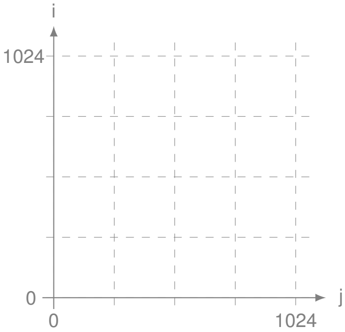
Polyhedral Abstraction



The Polyhedral Model

Input Loop Nest

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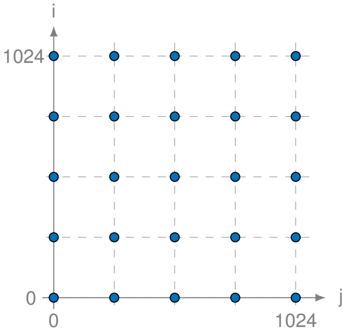

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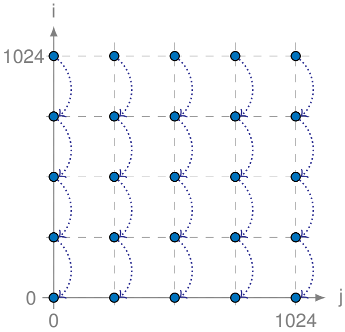

The Polyhedral Model

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Input Loop Nest

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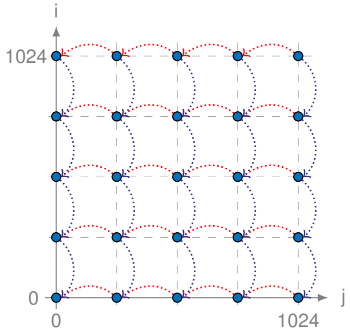
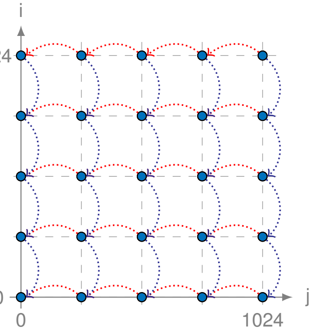
Polyhedral Abstraction



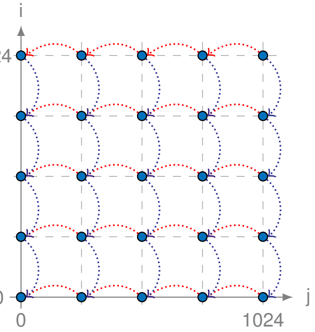
The Polyhedral Model

Original

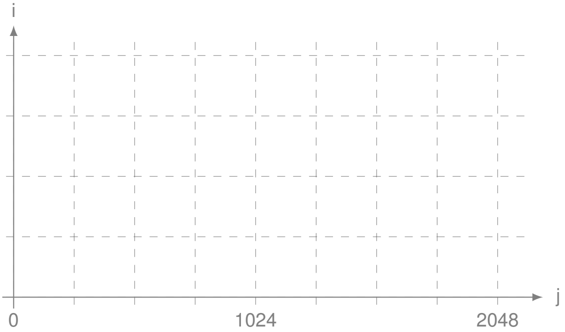


The Polyhedral Model

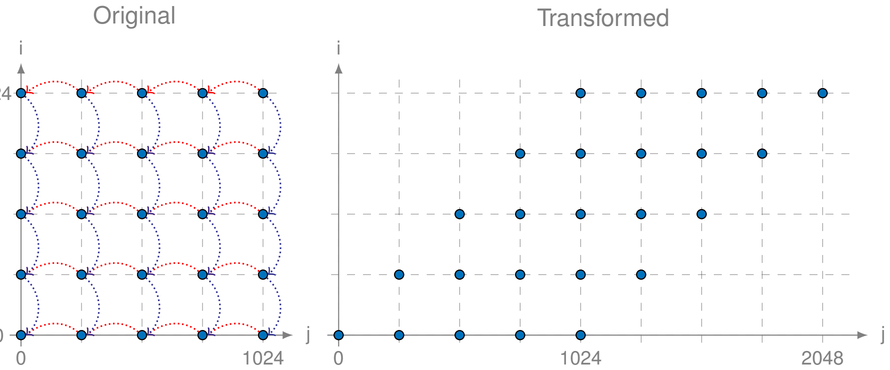
Original



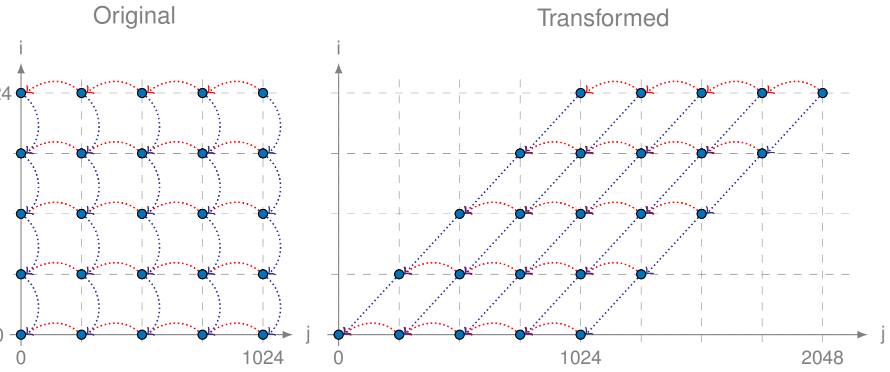
Transformed



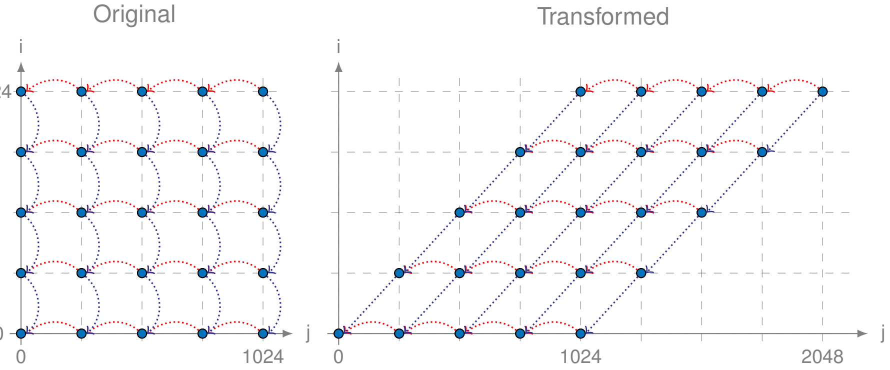
The Polyhedral Model



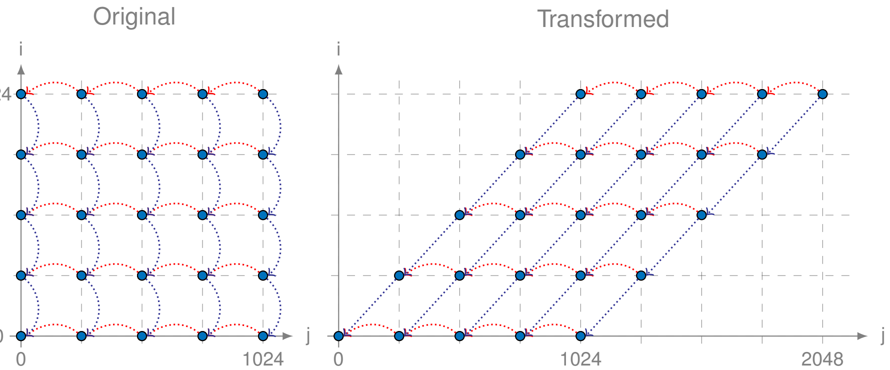
The Polyhedral Model



The Polyhedral Model



The Polyhedral Model




```
for (j = 0; j <= 2048; j++) {
  parfor (i = max(j - 1024, 0); i <= min(j, 1024); i++) {
    s[j - i] = s[j - i] + r[i] * A[i][j - i];
    q[i] = q[i] + A[i][j - i] * p[j - i];
  }
}
```

The Polyhedral Model

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Input Loop Nest

The Polyhedral Model

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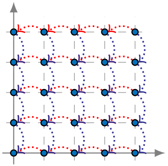
Input Loop Nest

Static Analysis

Polyhedral Abstraction



The Polyhedral Model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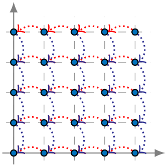
Input Loop Nest

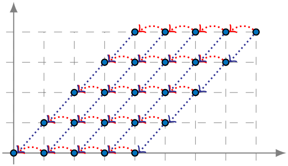
Static Analysis

Polyhedral Abstraction

Scheduler

Schedule





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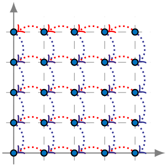
Input Loop Nest

Static Analysis

Polyhedral Abstraction



Scheduler

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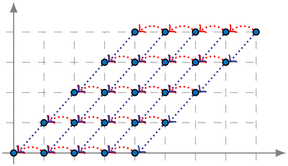

Code Generation

Output Loop Nest

```
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    s[j - i] = s[j - i] + r[i] * A[i][j - i];
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```